Toorikkonspekt kursuse “Biofüüsika” füüsikalistest küsimustest

5 . VOOLAMINE
FLOW

KÄESOLEVAS FAILIS sisaldub konspekti toorik aine “Biofüüsika” **järgmiste osade jaoks:**

Vedelike ja gaaside voolamine

THE PRESENT FILE contains provision for the course “Biophysics” **corresponding to the following parts of the course:**

Flows of Liquids and Gases

Clapeyron-Mendelejevi seadus

$$pV = \frac{m}{\mu} RT$$

Ideaalne gaas
Ideal gas
Идеальный газ

p – rõhk / pressure / давление, Pa

V – ruumala / volume / объем, m^3

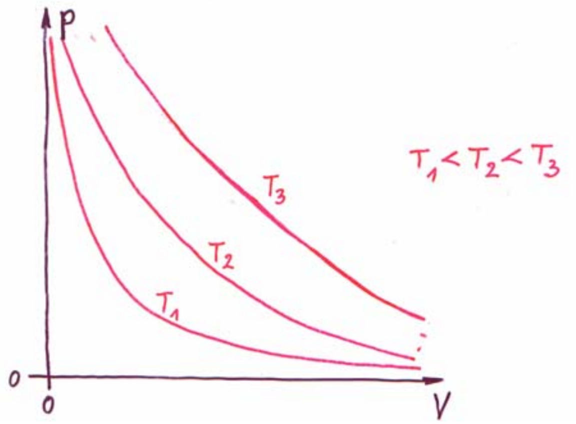
m – mass / mass / масса, kg

μ – moolimass / molar mass / молярная масса, $\frac{kg}{mol}$

T – absoluutne temp. / absolute temper. / абсолютная темп., K

Universaalne gaasi konstant:
Universal gas constant:
Универсальная газовая постоянная:

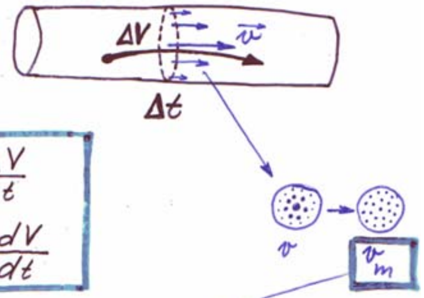
$$R = 8{,}31 \frac{J}{mol \cdot K}$$

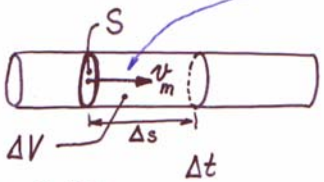


Voolamise mahtkiirus:
Volume flow:
Расход:

$$\bar{Q} \stackrel{\Delta}{=} \frac{\Delta V}{\Delta t}$$

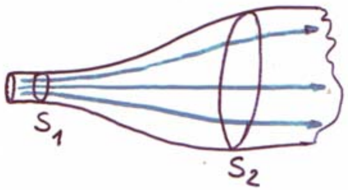
$$Q(t) \stackrel{\Delta}{=} \frac{dV}{dt}$$





$$\bar{Q} = \frac{\Delta V}{\Delta t} = \frac{S \cdot \Delta s}{\Delta t} = S \cdot \bar{v}_m$$

$$Q = S \cdot v_m$$



$$Q_1 = Q_2$$

$$S_1 \cdot v_{m1} = S_2 \cdot v_{m2}$$

$$\frac{v_{m1}}{v_{m2}} = \frac{S_2}{S_1}$$

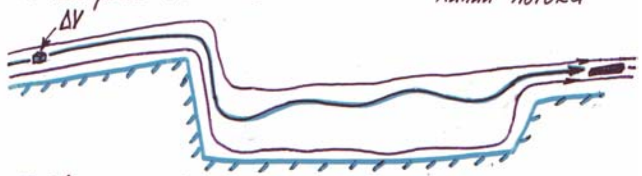
Vedeliku voolamise mudel inertsi arvestamisega ja sisehõõrdumist arvestamata.

Model of liquid flow that takes account of inertance and neglects viscosity.

Модель течения жидкости, учитывающая инерцию и пренебрегающая вязкостью.

Laminaarne voolamine
Laminar flow
Ламинарное течение

Voolujooned
Streamlines
Линии потока



Eeldus: Puuduvad dissipatiivsed jõud
Presumption: There are no dissipative forces
Предпосылка: Отсутствуют диссипативные силы

ΔV:

Kogu teekonnal kehtib:
On the whole travel is valid:
На всем пути имеет место:

$$E_K + E_P = const$$

$$E_K = \frac{\Delta m \cdot v^2}{2} = \frac{\rho \Delta V \cdot v^2}{2}$$

$$E_P = \Delta m \cdot g \cdot h + p \cdot \Delta V = \rho \Delta V \cdot g h + p \cdot \Delta V$$

$$\frac{\rho v^2}{2}\cdot\Delta V + \rho g h\cdot\Delta V + p\cdot\Delta V = const \;\Big|\, :\Delta V$$

$$\frac{\rho v^2}{2} + \rho g h + p = const_1 \equiv p_0$$

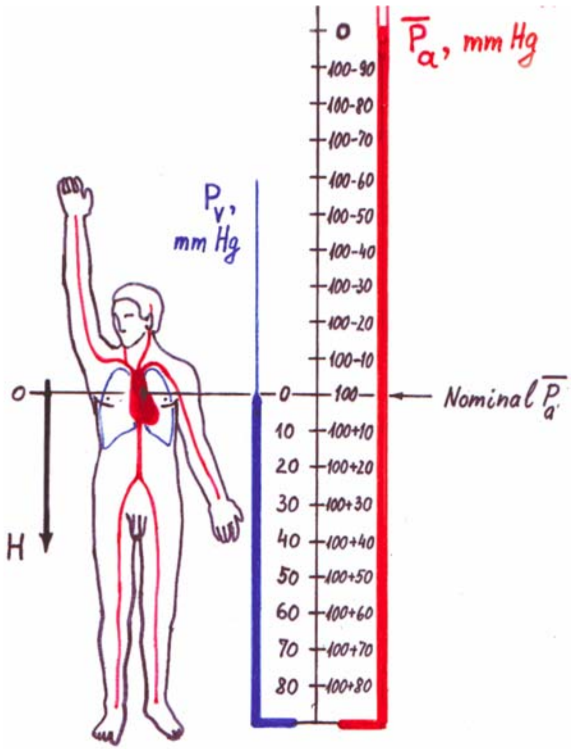
$$\boxed{\frac{\rho v^2}{2} + \rho g h + p = p_0}$$

Bernoulli võrrand
Bernoulli equation
Уравнение Бернулли

$$\boxed{p_0 = p\Big|_{\substack{v=0\\ h=0}}}$$

$h=0$: $\quad \frac{\rho v^2}{2} + p = p_0 \;\longrightarrow\; \boxed{\Delta p_{dyn} \triangleq p - p_0 = -\frac{\rho v^2}{2}}$

$v=0$: $\quad \rho g h + p = p_0 \;\longrightarrow\; \boxed{\Delta p_{gr} \triangleq p - p_0 = -\rho g h = \rho g H}$



$$\overline{P}_a = \overline{P}_a\Big|_{H=0} + \rho g H$$

$$P_v = P_v\Big|_{H=0} + \rho g H$$

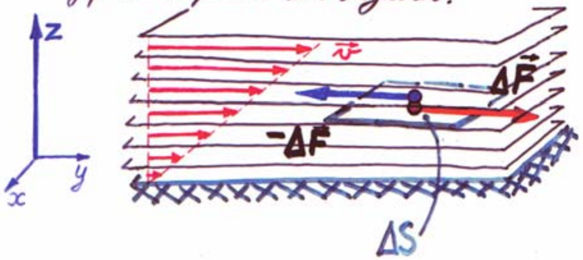
3.5.5

II: Vedeliku voolamise mudel sisehõõrdumise arvestamisega ja inertsi arvestamata.

Model of liquid flow that takes account of viscosity and neglects inertance.

Модель течения жидкости, учитывающая вязкость и пренебрегающая инерцией.

Sisehõõrdumine e. viskoossus:
Viscosity (internal friction):
Внутреннее трение или вязкость:



$$\frac{\Delta F}{\Delta S} = \eta \cdot \frac{dv}{dz}$$

Newtoni viskoossuse-võrrand
Newton's equation for viscosity
Уравнение вязкости Ньютона

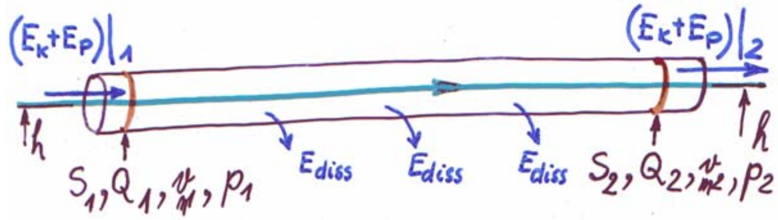
Viskoossuse kordaja
Viscosity coefficient η:
Коэффициент вязкости

$$[\eta] = \left[\frac{F}{S} \cdot \frac{z}{v}\right] = \frac{1N}{1m^2} \cdot \frac{1m}{1\frac{m}{s}} = 1\,Pa \cdot s$$

Laminaarne viskoosne voolamine torudes:
Laminar viscous flow in tubes:
Ламинарное вязкое течение в трубах:

NB! Dissipatiivsed jõud
Dissipative forces $\Rightarrow$ $E_K + E_P \neq const$
Диссипативные силы

$$\frac{\rho v^2}{2} + \rho g h + p \neq const$$



$$h = const, \quad \rho = const, \quad S_1 = S_2$$

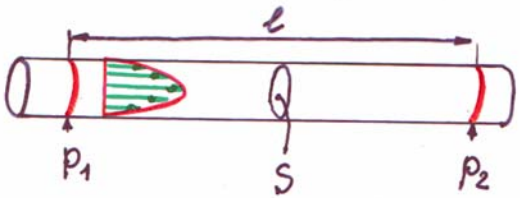
$$Q_1 = Q_2 \Rightarrow v_{m1} \cdot S_1 = v_{m2} \cdot S_2 \Rightarrow v_{m1} = v_{m2}$$

$$(E_K + E_P)|_1 > (E_K + E_P)|_2$$

$$\frac{\rho v^2}{2} + \rho g h + p_1 > \frac{\rho v^2}{2} + \rho g h + p_2$$

$$\underline{p_1 > p_2}$$

3.5.7



NB!
Horisontaalne
Horizontal
Горизонтально

$$Q = \frac{p_1 - p_2}{\mathcal{R}} , \quad \mathcal{R} = 8\pi \cdot \eta \cdot \frac{\ell}{S^2}$$

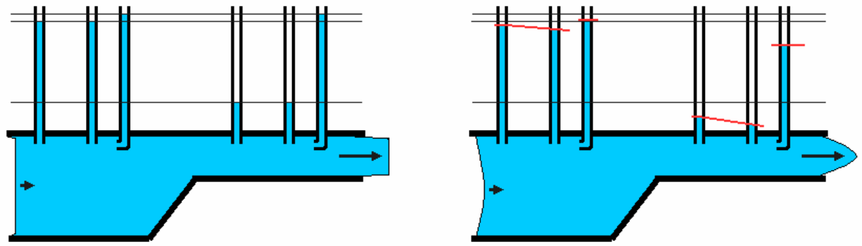
Poiseuille'i seadus
Poiseuille law
Закон Пуазейля

J.L.M. Poiseuille
1799 - 1869

$$[\mathcal{R}] = \left[\eta \cdot \frac{\ell}{S^2}\right] = 1 Pa \cdot s \cdot \frac{m}{m^4} = 1 \frac{Pa \cdot s}{m^3}$$

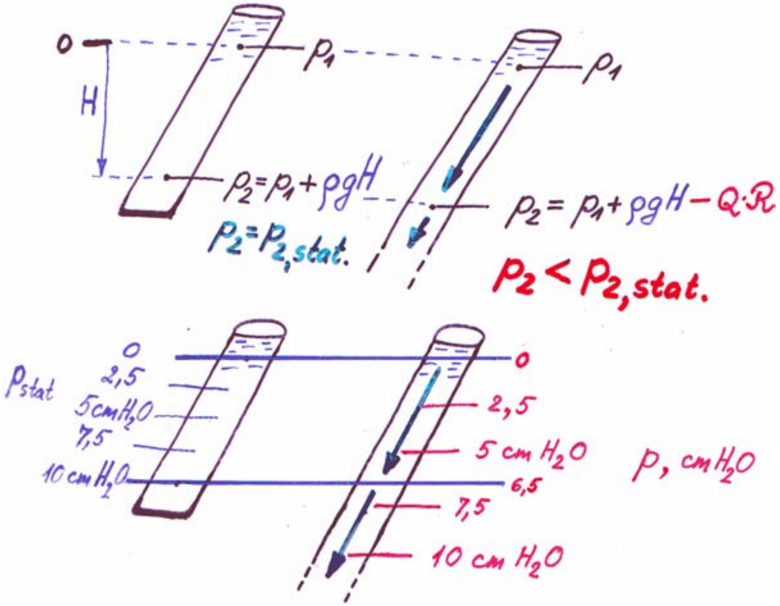
$$[\mathcal{R}] = \left[\frac{p}{Q}\right] = \frac{1 Pa}{1 \frac{m^3}{s}} = 1 \frac{Pa \cdot s}{m^3}$$

$\mathcal{R}$ – hüdrauliline takistus
hydraulic resistance
гидравлическое сопротивление



Ideaalne mitteviskoosne voolamine
(Bernoulli voolamine)
Idealized non-viscous flow
(Bernoulli flow)
Идеальное безвязкостное течение
(течение Бернулли)

Viskoosne voolamine
(Poiseuille'i voolamine)
Viscous flow
(Poiseuille flow)
Вязкостное течение
(течение Пуазейля)

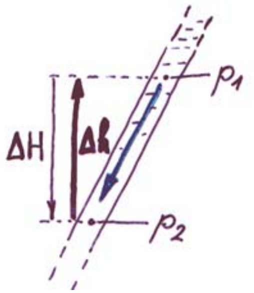


$$Q = \frac{(p_1 + \rho g H_1) - (p_2 + \rho g H_2)}{\mathcal{R}} = \frac{(p_1 - p_2) + \rho g (H_1 - H_2)}{\mathcal{R}}$$

$$\Delta h = -(H_1 - H_2)$$

$$\boxed{Q = \frac{(p_1 - p_2) + \rho g \cdot \Delta h}{\mathcal{R}}}$$

Üldjuhtum
General case
Общий случай

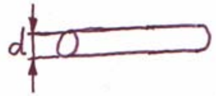


III : Turbulents / Turbulence / Турбуленция — Keerised / Vortices, whirls / Завихрения, вихри

Reynoldsi arv (kriteerium):
Reynolds number:
Число Рейнольдса:

O. Reynolds 1842–1912

$$\boxed{Re = \frac{\rho v d}{\eta}}$$



← $Re_{kr} = 2300$

Turbulents: $\mathcal{R} \nearrow$

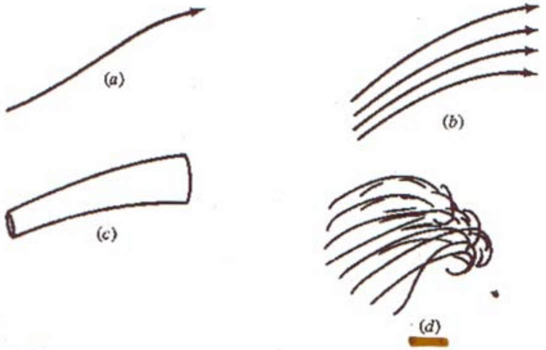


Figure (a) A single streamline. (b) A group of adjacent streamlines. (c) A flow tube. The walls of the tube are composed of streamlines. (d) Turbulent flow.

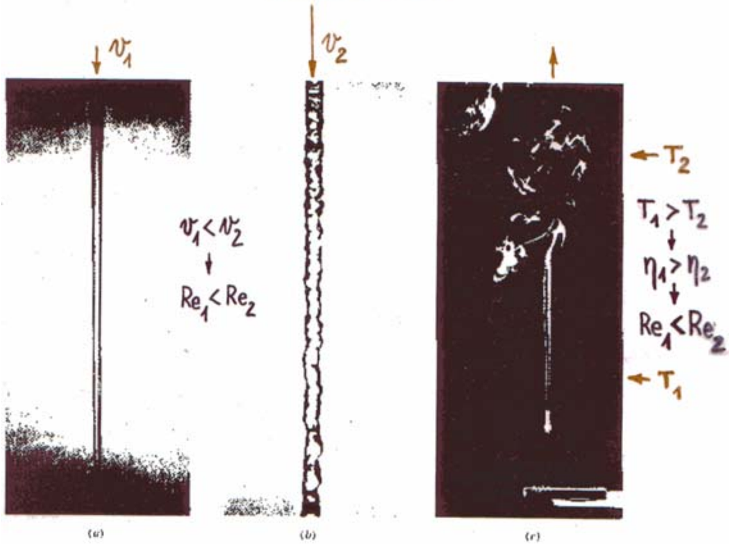
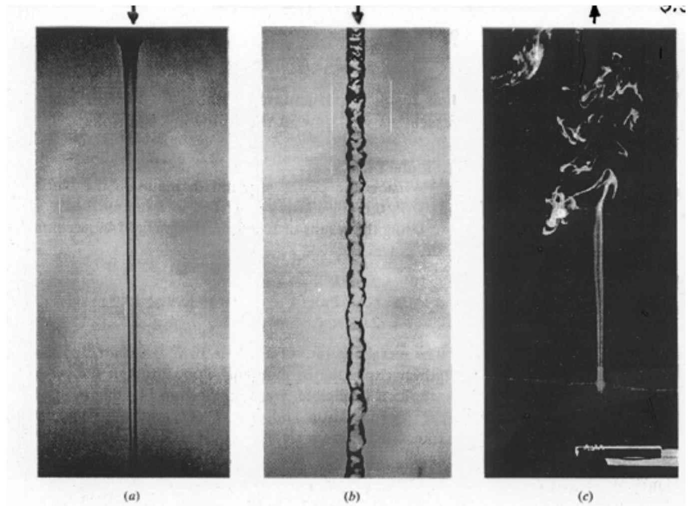


Figure 14.5. (a) Laminar flow and (b) turbulent flow of water. (c) First laminar, then turbulent flow of cigarette smoke. (From: Sears, Zemansky and Young, *University Physics*, fifth edition, Copyright © 1976, Addison-Wesley, Reading, Mass.)



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